Solutions to Problem 1.

a. $$\Pr\{Y=0\} = \Pr\{Y=0 \text{ and } X=1\} + \Pr\{Y=0 \text{ and } X=2\} + \Pr\{Y=0 \text{ and } X=3\}$$
$$= \frac{1}{3} + \frac{1}{4} + \frac{3}{16} = \frac{37}{48} \approx 0.7708$$

b. $$\Pr\{Y=1 \mid X=2\} = \frac{\Pr\{Y=1 \text{ and } X=2\}}{\Pr\{X=2\}} = \frac{\Pr\{Y=1 \text{ and } X=2\}}{\Pr\{Y=0 \text{ and } X=2\} + \Pr\{Y=1 \text{ and } X=2\} + \Pr\{Y=2 \text{ and } X=2\}}$$
$$= \frac{\frac{1}{12}}{\frac{1}{4} + \frac{1}{12} + 0} = \frac{1}{4}$$

c. $\Pr\{X=1 \text{ and } Y=2\}$ is the probability that Professor Right is asked 1 question and answers 2 questions incorrectly, which is impossible.

Solutions to Problem 2.

a. These probabilities are given to us in the problem:

$$\Pr\{M=1\} = 0.20 \qquad \Pr\{M=2\} = 0.30 \qquad \Pr\{M=3\} = 0.50$$

b. These probabilities are given to us in the problem:

$$\Pr\{D=1 \mid M=1\} = 0.01 \qquad \Pr\{D=1 \mid M=2\} = 0.02 \qquad \Pr\{D=1 \mid M=3\} = 0.03$$

c. Using the law of total probability:

$$\Pr\{D=1\} = \Pr\{D=1 \mid M=1\}\Pr\{M=1\} + \Pr\{D=1 \mid M=2\}\Pr\{M=2\} + \Pr\{D=1 \mid M=3\}\Pr\{M=3\}$$
$$= 0.01(0.20) + 0.02(0.30) + 0.03(0.50) = 0.023$$

Solutions to Problem 3.

a. First, let's compute

$$\Pr\{Z=2\} = \Pr\{Z=2 \text{ and } M=0\} + \Pr\{Z=2 \text{ and } M=1\} + \Pr\{Z=2 \text{ and } M=2\} = 0.25$$

The conditional probabilities of M given $Z=2$ are:

$$\Pr\{M=0 \mid Z=2\} = \frac{\Pr\{M=0 \text{ and } Z=2\}}{\Pr\{Z=2\}} = \frac{0.10}{0.25} = \frac{2}{5}$$
$$\Pr\{M=1 \mid Z=2\} = \frac{\Pr\{M=1 \text{ and } Z=2\}}{\Pr\{Z=2\}} = \frac{0.08}{0.25} = \frac{8}{25}$$
$$\Pr\{M=2 \mid Z=2\} = \frac{\Pr\{M=2 \text{ and } Z=2\}}{\Pr\{Z=2\}} = \frac{0.07}{0.25} = \frac{7}{25}$$

b. $$E[M \mid Z=2] = 0 \cdot \Pr\{M=0 \mid Z=2\} + 1 \cdot \Pr\{M=1 \mid Z=2\} + 2 \cdot \Pr\{M=2 \mid Z=2\} = \frac{22}{25}$$

c. M and Z are <u>not</u> independent: if they were, we would have $\Pr\{M=1\} = \Pr\{M=1 \mid Z=3\}$.

Solutions to Problem 4.

a.

$$\begin{aligned}\Pr\{X = 4 \mid X \neq 1\} &= \frac{\Pr\{X = 4 \text{ and } X \neq 1\}}{\Pr\{X \neq 1\}} \\ &= \frac{\Pr\{X = 4\}}{\Pr\{X \neq 1\}} \\ &= \frac{0.1}{0.3 + 0.5 + 0.1} = \frac{1}{9}\end{aligned}$$

b.

$$\begin{aligned}\Pr\{X = 4 \mid X \neq 1 \text{ and } X \neq 2\} &= \frac{\Pr\{X = 4 \text{ and } X \neq 1 \text{ and } X \neq 2\}}{\Pr\{X \neq 1 \text{ and } X \neq 2\}} \\ &= \frac{\Pr\{X = 4\}}{\Pr\{X \neq 1 \text{ and } X \neq 2\}} \\ &= \frac{0.1}{0.5 + 0.1} = \frac{1}{6}\end{aligned}$$

c.

$$\begin{aligned}\Pr\{X = 2 \mid X \leq 2\} &= \frac{\Pr\{X = 2 \text{ and } X \leq 2\}}{\Pr\{X \leq 2\}} \\ &= \frac{\Pr\{X = 2\}}{\Pr\{X \leq 2\}} \\ &= \frac{0.3}{0.1 + 0.3} = \frac{3}{4}\end{aligned}$$

Solutions to Problem 5.

a.

$$\begin{aligned}\Pr\{X_2 = 1 \mid X_1 = 0\} &= \frac{\Pr\{X_2 = 1 \text{ and } X_1 = 0\}}{\Pr\{X_1 = 0\}} \\ &= \frac{0.05}{0.80} = 0.0625\end{aligned}$$

b.

$$\begin{aligned}\Pr\{X_2 = 1 \mid X_1 = 1\} &= \frac{\Pr\{X_2 = 1 \text{ and } X_1 = 1\}}{\Pr\{X_1 = 1\}} \\ &= \frac{0.10}{0.20} = 0.50\end{aligned}$$

c. X_1 and X_2 are dependent because $\Pr\{X_2 = 1 \mid X_1 = 0\} \neq \Pr\{X_2 = 1 \mid X_1 = 1\}$.

d.

$$\begin{aligned}\text{expected profit} &= 100 \Pr\{X_2 = 0 \mid X_1 = 1\} + (-20) \Pr\{X_2 = 1 \mid X_1 = 1\} \\ &= 100(1 - 0.50) - 20(0.50) = 40\end{aligned}$$